HOMOTHERMAL SHOCK CAUSED BY THE ACTION OF INSTANTANEOUS MONOCHROMATIC RADIATION

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The self-similar motion of a gas heated by an instantaneous point isotropic source of monochromatic radiation was considered in [1] in an adiabatic approximation. However, in the case of strong heating of the gas the influence of the intrinsic radiation on the motion becomes substantial.

The influence of radiation is taken into account in this paper within the framework of a homothermal model corresponding to large values of the heat conduction coefficient [2].

At the initial time let the gas internal energy, velocity, and density satisfy the relationships

$$\varepsilon(r,\ 0) = A/r^2,\ v(r,\ 0) = 0,\ \rho(r,\ 0) = \rho_0. \tag{1}$$

The initial state of the gas, described by (1), can be obtained because of instantaneous liberation of the energy E_0 in a cold gas with the density ρ_0 by monochromatic radiation with the path length L [1]. In this case $\varepsilon(r,\ 0) = E_0 e^{-r/L}/4\pi\rho_0 L r^2$ and agrees with (1) ($A = E_0/4\pi\rho_0 L$) in the limit $r \ll L$.

Gas motion starts at $t > 0$. We assume that a strong shock whose front radiates an energy flux equilibrating the temperature in the flow domain [3] is propagated from the center of symmetry.

The system of equations describing the one-dimensional motion under consideration has the form

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0,\ \frac{\partial T}{\partial r} = 0,$$
$$\frac{\partial \rho}{\partial t} + v\frac{\partial \rho}{\partial r} + \rho\left[\frac{\partial v}{\partial r} + 2\frac{v}{r}\right] = 0.$$

Using the equation of state of an ideal gas $p = \rho RT$, and eliminating the pressure, we obtain

$$\begin{gathered}\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} + RT\frac{\partial}{\partial r}(\ln\rho) = 0,\\ \frac{\partial}{\partial t}(\ln\rho) + v\frac{\partial}{\partial r}(\ln\rho) + \frac{\partial v}{\partial r} + 2\frac{v}{r} = 0.\end{gathered} \tag{2}$$

From the laws of conservation of momentum and mass of the gas, we have the following relationships on the discontinuity $r = r_1$:

$$\rho_1(v_1 - D) = -\rho_0 D,\ \rho_1(v_1 - D)^2 + R\rho_1 T = \rho_0 D^2, \tag{3}$$

where D is the velocity of the shock front. The magnitudes behind and in front of the shock, respectively, are denoted by the subscripts 1 and 0.

Let us note that the gas motion in front of the shock is not taken into account in this paper in contrast to [1].

The gas velocity is $v(0,\ t) = 0$ at the center of symmetry.

For $t > 0$ the gas motion is self-similar. Let us introduce the self-similar variable

$$x = \beta r/r_1, \tag{4}$$

where $r_1 = \xi A^{1/4} t^{1/2}$, ξ, β are constants to be determined.

The formulas

$$v = \frac{D}{\beta}f,\ \rho = \rho_0 g,\ T = \frac{D^2}{\beta^2 R}, \tag{5}$$

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can be written for the gas velocity, density, and temperature, where $D = dr_1/dt$.

Substituting (4) and (5) into (2), we obtain a system of ordinary differential equations

$$\frac{d}{dx}(\ln g) = (x-f)\frac{df}{dx} + f, \quad \frac{df}{dx} = (x-f)\frac{d}{dx}(\ln g) - \frac{2f}{x}. \tag{6}$$

The first equation in (6) has an integral, but by eliminating $(d/dx)(\ln g)$ from the second, we find an equation to determine the gas velocity f:

$$\frac{df}{dx} = \frac{f}{x}\frac{x(x-f)-2}{1-(x-f)^2}, \quad g = Ce^{fx-f^2/2}, \tag{7}$$

where C is a constant of integration.

Going over to dimensionless coordinates for $x = \beta$ in (3), we have relationships for the functions f(x) and f(x) on the discontinuity

$$f_1 = (\beta + \sqrt{\beta^2 - 4})/2, \quad g_1 = \beta/(\beta - f_1), \tag{8}$$

while at the center of symmetry

$$f(0) = 0. \tag{9}$$

If the solution of (7) is determined which satisfies (8) and (9), then the whole gas motion can be computed.

Let us analyze the field of integral curves of (7) by using the results in [2]. The equation has singularities (Fig. 1) in the domain (x, f). The point O (0, 0) is a saddle point. The integral curves, the lines $x = 0$ and $f = 0$, enter it. The singularity A(1, 0) is a node, where the integral curve $f = 0$ enters the point A; the second integral curve entering the point A has the slope $k = 1/2$. We have a saddle point at the singularity B(2, 1) into which two integral curves with the slopes $k_1 = (1 + \sqrt{13})/4$, $k_2 = (1 - \sqrt{13})/4$ enter.

The curves $f = x - 1$ and $x = 0$ (the line of the infinite derivative) are presented dashed in Fig. 1, while $f = 0$ and $f = x - 2/x$ (the line of the zero derivative) separating the domains where the derivative df/dx changes sign are presented by dash-dot curves.

It follows from an analysis that the desired integral curve passes through the singularities O, A, and B, where it leaves from the point A with the slope $k = 1/2$ (curve 1 in Fig. 1).

The location of the shock front is determined by the intersection with the curve $f_1(x) = (x + \sqrt{x^2 - 4})/2$ (curve 2 in Fig. 1). The numerical solution of (7) shows that the intersection has the coordinates (2, 1), i.e., agrees with point B.

Computed profiles of the gas velocity and density are shown by the solid curve in Figs. 2 and 3, and (for comparison) analogous dependences for the adiabatic case with the adiabatic index $\gamma = 1.1$ by dashes. Let us note that the gas density and velocity profiles do not depend explicitly on γ for the homothermal case.

The substantial influence of the intrinsic gas radiation on its motion is seen from the results represented.

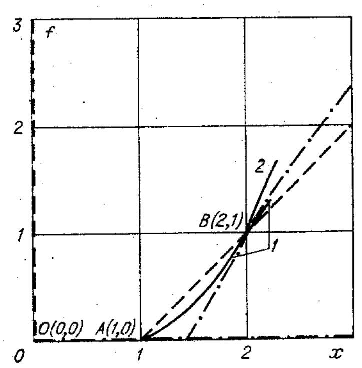

Fig. 1

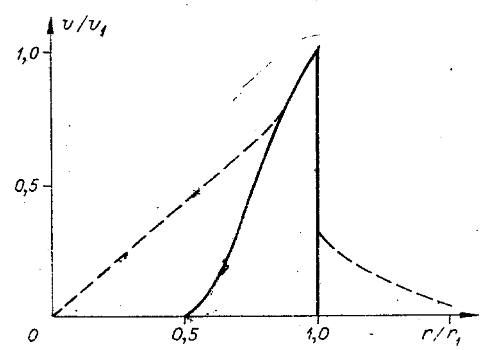

Fig. 2

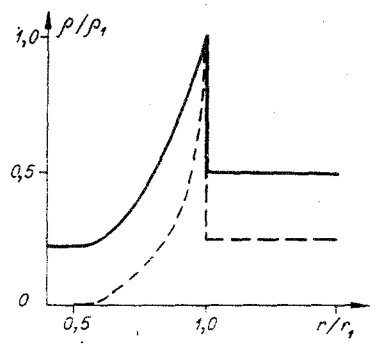

Fig. 3

The constant ξ can also be determined by using the energy conservation law. Let us assume that the energy of a gas heated by radiation in a domain bounded by r_1 is consumed by its motion:

$$4\pi \int_0^{r_1} \varepsilon(r, 0) \rho r^2 dr = 4\pi \int_0^{r_1} \left(\frac{\rho v^2}{2} + \frac{\rho R T(t)}{\gamma - 1} \right) r^2 dr. \quad (10)$$

Substituting the solution found into (10) and taking $\gamma = 1.1$ we obtain the value $\xi = 1.17$.

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LITERATURE CITED

1. I. G. Zhidov and V. G. Rogachev, "Self-similar motion of a gas heated by a point isotropic source of monochromatic radiation," Zh. Prikl. Mekh. Tekh. Fiz., No. 4 (1976).
2. V. P. Korobeinikov, Problems of the Theory of a Point Explosion in a Gas [in Russian], Nauka, Moscow (1973).
3. L. P. Gorbachev and V. F. Fedorov, "On the influence of a shock on heat propagation," Zh. Prikl. Mekh. Tekh. Fiz., No. 3 (1975).